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RESEARCH ARTICLE

A Survey on Weak Pseudoorders in Ordered Hyperstructures

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Abstract: As a generalization of pseudoorders, the weak pseudoorder in ordered (semi)hyperrings was defined by Qiang et al., and some results were studied. In order to further study, we apply weak pseudoorders for an ordered superring R and show relations with pseudoorders of R. Moreover, we present some illustrative examples and regular equivalence relation σ on ordered superring R, such that R/σ is an ordered superring. Furthermore, we show that if η is a weak pseudoorder on an ordered superring R, F is the set of all weak pseudoorders on R/η^* and $E = \{\zeta \mid \zeta$ is a weak pseudoorder on R such that $\eta \subseteq \zeta\}$, then card(E) = card(F).

keywords: ordered superring, weak pseudoorder, regular relation

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1. Introduction

Marty (1934) and Krasner (1983) introduced the hypergroups and Krasner hyperrings as a generalization of groups and rings. For the basic definitions, terminology and applications of hyperstructures, the reader is referred to the fundamental books (Corsini, 1993; Corsini & Leoreanu, 2003; Davvaz & Leoreanu-Fotea, 2007; Davvaz & Vougiouklis, 2019; Vougiouklis, 1994). Vougiouklis (1990) introduced the definition of semihyperrings and proved some basic results. Algebraic geometry over hyperrings was investigated by Jun (2018). Asokkumar (2013) extended derivations to prime hyperrings. Hila et al. (2018) explored some characterizations of Krasner (m,n)-hyperrings through their (k,n)-absorbing hyperideals.

Heidari and Davvaz (2011) investigated the notion of ordered hyperstructures. For the first time, the idea of pseudoorders in ordered semigroups was presented by Kehayopulu and Tsingelis (1995a, 1995b). Then, this concept was moved to ordered semihypergroups by Davvaz et al. (2015). Feng et al. (2018) studied pseudoorders in ordered *-semihypergroups. Finally, it was extended to ordered (semi) hyperrings by Omidi and Davvaz (2016, 2017). They investigated the notions of strongly regular relations of an ordered hyperstructure and applied them to construct an ordered structures. The connection between ordered semihypergroups was established by Gu and Tang (2016); Tang et al. (2018). For the work done on ordered semihyperrings, we point out to Omidi and Davvaz (2018).

Rao et al. (2022) characterized ordered Γ-semihypergroups based on their weak Γ-hyperfilters. In addition, in Qiang et al. (2021), w-pseudo-orders in ordered (semi)hyperrings are given. Rao, Zhao, et al. (2021); Rao, Kosari, et al. (2021) introduced some new concepts of ordered hyperstructures. Shi et al. (2021) introduced the notion of a factorizable ordered hypergroupoid and discussed some related properties. The fuzzy interior hyperideals (Tipachot & Pibaljommee, 2016); the (m,n)-hyperideals (Mahboob et al., 2020); the uni-soft interior Γ-hyperideals (Khan et al., 2020) and the prime (m,n) bi-Γ-hyperideals (Yaqoob & Aslam, 2014) have been introduced and investigated. Khan et al. (2020) characterized ordered hyperstructures in terms of their uni-soft hyperideals. Ameri and Hedayati (2007) gave the definition and examples of k-hyperideals in semihyperrings.

We are fully aware that an ordered structure such as an ordered semigroup has a very close relation with the theory of decision processes, artificial intelligence, information retrieval, etc. We are eager to study the weak pseudoorders in an ordered hyperstructures. We apply the concept of the weak pseudoorder for an ordered superring. The concepts have been supported by

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illustrative examples on ordered hyperstructures. Also, we present some results connected with the weak pseudoorder.

2. Preliminaries

Let R, Γ ≠ ∅. If

(1) every γ ∈ Γ is a hyperoperation on R,
(2) for every α, β ∈ Γ and a, b, c ∈ R, we have aα(bβc) = (aαb)βc,

then R is a Γ-semihypergroup. If ∅ ≠ U, V ⊆ R, then

$$U\Gamma V = \bigcup_{\gamma\in\Gamma} U\gamma V = \bigcup \{u\gamma v \mid u \in U; v \in V \text{ and } \gamma \in \Gamma\}.$$

Definition 2.1. (R,+, ·) is a superring (Ameri et al., 2019) if ∀t,q,m ∈ R,

(1) (R,+) is a canonical hypergroup;
(2) (R, ·) is a semihypergroup s.t., t · 0 = 0 = 0 · t;
(3) t · (q+m) = t · q + t · m and (q+m) · t = q · t + m · t;
(4) q · (−m) = (−q) · m = −(q · m).

(R,+, ·) is a hyperring (Krasner, 1983; Davvaz & Leoreanu-Fotea, 2007) if it satisfies (1), (3) and (2) and (R, ·) is a semigroup s.t., m · 0 = 0 = 0 · m, ∀m ∈ R.

Definition 2.2. (R,+, ·,≲) is an ordered superring (hyperring) if

(1) (R,+, ·) is a superring (hyperring);
(2) (R,≲) is a poset;
(3) (∀t, q, m∈R) q ≲ m implies q + t ≼ m + t and t + q ≼ t + m;
(4) (∀t, q, m∈R) q ≲ m implies q · t ≼ m · t and t · q ≼ t · m.

Here, a relation J ≼ G is only possible if ∀j ∈ J, ∃g ∈ G s.t., j ≲ g, where ∅ ≠ J, G ⊆ R.

We set

(1) $E\overrightarrow{\sigma}F$ ⇔ yt ∈ E; 3q ∈ F; tσq.
(2) $E\overleftarrow{\sigma}F$ ⇔ ∀f' ∈ F; 3e' ∈ E; e'σf'.
(3) EeσF ⇔ y t ∈ E; 3q ∈ F; tσq and q σ t & ∀f' ∈ F, ∃e' ∈ E; f'σ e' and e' σf'.
(4) EσF ⇔ $E\overrightarrow{\sigma}F$ and $E\overleftarrow{\sigma}F$.
(5) EσF ⇔ ye ∈ E; yf ∈ F; eσf.

Definition 2.3. σ is a pseudoorder on R if ∀t, q, w ∈ R

(1) ≤ ⊆ σ;
(2) tσq and qσw ⇒ tσw;
(3) qσw ⇔ (q + t)σ(w + t) and (t + q)σ(t + w);
(4) qσw ⇔ (q . t)σ(w . t) and (t . q)σ(t . w).

3. Weak Pseudoorder on Ordered Superrings

Weak pseudoorder on an ordered hyperstructure was investigated by Tang et al. (2018), Rao, Kosari, et al. (2021) and Qiang et al. (2021). Clearly, ≤ is a weak pseudoorder, and also every pseudoorder relation is a weak pseudoorder.

Example 1. Let R = {0,t, q, m} and

Table 1
hyperoperation +

+	0	t	q	w
0	0	t	q	w
t	t	{0, q}	{t, w}	q
q	q	{t, w}	{0, q}	t
w	w	q	t	0

Table 2
operation ·

·	0	t	q	w
0	0	0	0	0
t	0	t	q	w
q	0	q	q	0
w	0	w	0	w

$$\leq := \{(0, 0), (t, t), (q, q), (w, w), (0, q), (w, t)\}.$$

It can be seen that

$$\sigma = \{(0; 0); (0;q); (t; t); (t; m); (q; 0); (q;q); (m; t); (m; m)\}$$

is a pseudoorder on ordered hyperring (R,+, ·,≲). Clearly, R/σ = {o₁, o₂}, where o_1 = {0, q} and o_2 = {t, m} and (R/σ, ⊕, ⊙, $\preccurlyeq_R$) are an ordered ring, where

Table 3
operation ⊕

⊕	k_1	k_2
k_1	k_1	k_2
k_2	k_2	k_1

Table 4
operation ⊙

⊙	k_1	k_2
k_1	k_1	k_1
k_2	k_1	k_2

and

$$\preceq_R = \{(o_1; o_1); (o_2; o_2)\}.$$

Definition 3.1. ∅ ≠ M ⊆ R is a hyperideal of an ordered superring (hyperring) (R,+, ·,≲) if

(1) m, m' ∈ M ⇒ m + m' ⊆ M and −m ∈ M;

(2) $q \in R \Rightarrow q \, . \, m, m \, . \, q \leq M$;
(3) $m \in M$, $q \in R$ and $q \leq m \Rightarrow q \in M$.

In condition (1), if $M + R \leq M$, then M is called a 2-hyperideal of R.

Question. Is there an ordered regular relation η on an ordered superring $(R,+,.,\leq)$ for which R/η is an ordered superring?

Omidi and Davvaz (2018) only provided a partial answer to the above problem in the context of ordered semihyperrings by using proper 2-hyperideals. However, for an ordered hyperring (superring) R, R does not necessarily exist a proper 2-hyperideal. In the following, we illustrate this concept with an example (see Example 2).

Example 2. Let $R = \{0, t, q, m\}$ and

Table 5
hyperoperation $+^{/}$

+	0	t	q	w
0	0	t	q	w
t	t	{0, t}	w	{q, w}
q	q	w	{0, q}	{t, w}
w	w	{q, w}	{t, w}	R

Table 6
operation $.^{/}$

·	0	t	q	w
0	0	0	0	0
t	0	{0, t}	0	{0, t}
q	0	0	{0, q}	{0, q}
w	0	{0, t}	{0, q}	{0, w}

Then, $(R,+^{/},.^{/})$ is a superring (Ameri et al., 2019). By setting

$$I \leq := \{(0;0); (t;t); (q;q); (m;m); (0;t); (q;m)\};$$

$(R,+^{/},.^{/},\leq)$ is an ordered superring. Let $A = \{M \mid M$ is a proper 2-hyperideal of $R\}$. We observe that $A = \emptyset$.

Now, we apply the idea of weak pseudoorder for an ordered superring in the following manner and give some examples.

Definition 3.2. σ is said to be a weak pseudoorder if $\forall e, f, q \in R$,

(1) $\leq \; \leq \sigma$;
(2) $e\sigma f$ and $f\sigma q \Rightarrow e\sigma q$;
(3) $e\sigma f \Leftrightarrow (e+q)\overrightarrow{\sigma}(f+q)$ and $(q+e)\overrightarrow{\sigma}(q+f)$;
(4) $e\sigma f \Leftrightarrow (e \, . \, q)\overrightarrow{\sigma}(f \cdot q)$ and $(q \cdot e)\overrightarrow{\sigma}(q \cdot f)$;
(5) $e\sigma f$ and $f\sigma e \Leftrightarrow (e+q)\tilde{\sigma}(f+q)$ and $(q+e)\tilde{\sigma}(q+f)$;
(6) $e\sigma f$ and $f\sigma e \Leftrightarrow (e \, q)\tilde{\sigma}(f \, . \, q)$ and $(q \, e)\tilde{\sigma}(q \, f)$.

Clearly, ≤ is a weak pseudoorder. and every pseudoorder relation is a weak pseudoorder.

Example 3. Let us continue with the ordered superring $(R,+^{/},.^{/},\leq)$ in Example 2. We set

$$I\sigma := \{(0;0); (t;t); (q;q); (m;m); (0;t); (t;0); (q;m); (m;q)\}.$$

Then, σ is a weak pseudoorder, but it is not a pseudoorder, since $(q +^{/} q)\sigma(m +^{/} q)$ does not hold. Indeed:

$$(q;m) \in \sigma; q +^{/} q = \{0;q\} \text{ and } m +^{/} q = \{t;m\} \text{ but } (0;m); (q;t) \notin \sigma$$

Example 4. In Example 3,

$$\sigma = \{(0;0); (0;t); (t;0); (t;t); (q;q); (q;w); (w;q); (w;w)\}.$$

is a weak pseudoorder. Clearly, $R/\sigma = \{o_1, o_2\}$, where $o_1 = \{0, t\}$ and $o_2 = \{q, m\}$. Also, $(R/\sigma, \boxplus^{/}, \ominus^{/}, \leq_R)$ is an ordered superring, where

Table 7
hyperoperation $\boxplus^{/}$

⊕	k_1	k_2
k_1	k_1	k_2
k_2	k_2	$\{k_1, k_2\}$

Table 8
hyperoperation $\ominus^{/}$

①	k_1	k_2
k_1	k_1	k_1
k_2	k_1	$\{k_1, k_2\}$

and

$$\leq_R = \{(o_1; o_1); (o_2; o_2)\}.$$

Now, let $e \leq f$ where $e, f \in R$. As σ is a weak pseudoorder, we obtain $(e,f) \in \; \leq \; \leq \sigma$. So, $\sigma^*(e) \leq \sigma^*(f)$.

Example 5. Let $R = \{t, q, w, r, s, t\}$ and $\Gamma = \{\gamma, \beta\}$. Define

Table 9
hyperoperation γ

γ	t	q	w	j	k	l
t	{t, q}	{t, q}	w	{j, k}	{j, k}	l
q	{t, q}	q	w	{j, k}	k	l
w	w	w	w	l	l	l
j	{j, k}	{j, k}	l	{t, q}	{t, q}	w
k	{j, k}	k	l	{t, q}	q	w
l	l	l	l	w	w	w

Table 10
hyperoperation β

β	t	q	w	j	k	l
t	$\{t, j\}$	$\{t, q, j, k\}$	$\{w, l\}$	$\{t, j\}$	$\{t, q, j, k\}$	$\{w, l\}$
q	$\{t, q, j, k\}$	$\{t, q, j, k\}$	$\{w, l\}$	$\{t, q, j, k\}$	$\{t, q, j, k\}$	$\{w, l\}$
w	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$
j	$\{t, j\}$	$\{t, q, j, k\}$	$\{w, l\}$	$\{t, j\}$	$\{t, q, j, k\}$	$\{w, l\}$
k	$\{t, q, j, k\}$	$\{t, q, j, k\}$	$\{w, l\}$	$\{t, q, j, k\}$	$\{t, q, j, k\}$	$\{w, l\}$
l	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$	$\{w, l\}$

$$\leq := \{(t, t), (t, q), (q, q), (w, w), (j, j), (j, k), (k, k), (l, l)\}.$$

Put

$$\|\sigma = \{(t; t); (t;q); (t; w); (q; t); (q;q); (q; w); (w; t); (w;q); (w; w); (j;j); (j; k); (j; l); (k;j); (k; k); (k; l); (l;j); (l; k); (l; l)\}.$$

Clearly, $R/\sigma = \{o_1, o_2\}$,where $o_1 = \{t, q, w\}$ and $o_2 = \{j, k, l\}$,is an ordered Γ_σ-semihypergroup, where

Table 11
hyperoperation γ_σ

γ_σ	z_1	z_2
z_1	z_1	z_2
z_2	z_2	z_1

Table 12
hyperoperation β_σ

β_σ	z_1	z_2
z_1	$\{z_1, z_2\}$	$\{z_1, z_2\}$
z_2	$\{z_1, z_2\}$	$\{z_1, z_2\}$

and

$$\leq_\sigma = \{(o_1; o_1); (o_2; o_2)\}.$$

Proposition 3.3. Let η be a weak pseudoorder on an ordered superring $(R,+,\cdot,\leq)$. Define

$$\eta^* = \{(p;q) \in R \times R \mid p\eta q \text{ and } q\eta p\}$$

an

$$\leq: = \{(\eta^*(e); \eta^*(f)) \in R/\eta^* \times R/\eta^* \mid \exists p \in \eta^*(e); \exists q \in \eta^*(f) \text{ such that } (p;q) \in \eta\}.$$

If

$$E = \{\zeta \mid \zeta \text{ is a weak pseudoorder on R and } \eta \leq \zeta\}$$

and

$$F = \{\zeta^l \mid \zeta^l \text{ is a weak pseudoorder on } R/\eta^*;$$

then card(E) = card(F).

Proof. For $\zeta \in E$, we set

$$\zeta^l := \{(\eta^*(x); \eta^*(y)) \in R/\eta^* \times R/\eta^* \mid \exists p \in \eta^*(x); \exists q \in \eta^*(y) \text{ such that } (p;q) \in \zeta\}.$$

Clearly,

$$(\eta^*(e); \eta^*(f)) \in \zeta^l \Leftrightarrow (e;f) \in \zeta.$$

Claim: ζ^l is a weak pseudoorder on R/η^*.

Let $(\eta^*(e),\eta^*(f)) \in \leqslant$. Then $(e,f) \in \eta \subseteq \zeta$. So, $(\eta^*(e),\eta^*(f)) \in \zeta^l$ and hence $\leqslant \subseteq \zeta^l$.

Now, let $(\eta^*(e),\eta^*(f)) \in \zeta^l$ and $(\eta^*(f),\sigma^*(g)) \in \zeta^l$. Then, $(e,f) \in \zeta$ and $(f,g) \in \zeta$. So, $(e,g) \in \zeta$. Thus, $(\eta^*(e),\eta^*(g)) \in \zeta^l$.

As ζ is a weak pseudoorder on R, we obtain $e + g \overrightarrow{\zeta} f + g$. So,

$$\forall m \in e + g; \exists n \in f + g \text{ such that } m\zeta n.$$

Hence, $(\eta^*(m),\eta^*(n)) \in \zeta^l$. Thus,

$$\eta^*(e) \boxplus \eta^*(g) = \bigcup_{m \in e+g} \eta^*(m) \overrightarrow{\zeta^l} \bigcup_{n \in f+g} \eta^*(n) = \eta^*(f) \boxplus \eta^*(g).$$

Similarly,

$$\eta^*(e) \odot \eta^*(g) \overrightarrow{\zeta^l} \eta^*(f) \odot \eta^*(g).$$

Let $(\eta^*(e),\eta^*(f)) \in \zeta^l$ and $(\eta^*(f),\eta^*(e)) \in \zeta^l$. Then, $(e,f) \in \zeta$, $(f,e) \in \zeta$. As ζ is a weak pseudoorder, $e + g \overrightarrow{\zeta} f + g$. Therefore, if $\zeta \in E$, then ζ^l is a weak pseudo order on R/η^*

Define $\varphi : E \to F$ by $\phi(\zeta) = \zeta^l$. Claim: ϕ is a bijection mapping. Step 1. ϕ is well-defined.

Let $\zeta_1; \zeta_2 \in E$, $\zeta_1 = \zeta_2$ and $(\eta^*(e),\eta^*(f)) \in \zeta_1{}^l$. Then, $(e,f) \in \zeta_1 = \zeta_2$ which implies that $(\eta^*(e),\eta^*(f)) \in \zeta_2{}^l$. Thus, $\zeta_1{}^l \subseteq \zeta_2{}^l$. Similarly, $\zeta_2{}^l \subseteq \zeta_1{}^l$. Step 2. ϕ is one to one.

Let $\phi(\zeta_1) = \phi(\zeta_2)$. Then, $\zeta_1{}^l = \zeta_2{}^l$. Let $(e,f) \in \zeta_1$ is an arbitrary element. Then, $(\eta^*(e),\eta^*(f)) \in \zeta_1{}^l$ and so $(\eta^*(e),\eta^*(f)) \in \zeta_2{}^l$. Thus, $(e,f) \in \zeta_2$. Thus, $\zeta_1 \subseteq \zeta_2$. Similarly, $\zeta_2 \subseteq \zeta_1$. Step 3. ϕ is onto.

Consider $\sigma \in F$ and

$$\zeta = \{(x;y) \in R \times R \mid (\eta^*(x); \eta^*(y)) \in \sigma\}.$$

Claim: $\zeta \in E$.

If $(t,q) \in \eta$, then $(\eta^*(t),\eta^*(q)) \in \leqslant \subseteq \sigma$, and so $(t,q) \in \zeta$. If $(t,q) \in \leq$, then $(t,q) \in \eta \subseteq \zeta$. Hence, $\leq \subseteq \zeta$. Let $(t,q) \in \zeta$ and

$(q,z) \in \zeta$. Then, $(\eta^*(t),\eta^*(q)) \in \sigma$ and $(\eta^*(q),\eta^*(z)) \in \sigma$. Thus, $(\eta^*(t),\eta^*(z)) \in \sigma$. This means that $(t,z) \in \zeta$. Now, let $(t,q) \in \zeta$ and $z \in R$. Then, $(\eta^*(t),\eta^*(q)) \in \sigma$ and $\eta^*(z) \in R/\eta^*$. As $\sigma \in F$, we obtain

$$\eta^*(t) \boxplus \eta^*(z) = \bigcup_{p \in t + z} \eta^*(p) \overrightarrow{\sigma} \bigcup_{m \in q + z} \eta^*(m) = \eta^*(q) \boxplus \eta^*(z).$$

So,

$$\forall p \in t + z; \exists m \in q + z \text{ such that } (\sigma^*(p); \sigma^*(m)) \in \sigma.$$

Hence, $(p,m) \in \zeta$ and thus $t + z \overrightarrow{\zeta} q + z$. Similarly, $z . t \overrightarrow{\zeta} z . q$. Therefore, $\zeta \in E$. Now, clearly $\phi(\zeta) = \zeta' = \sigma$. □

4. Conclusions

In this paper, the concept of weak pseudoorder is introduced for ordered superrings as an extension of the notion of pseudoorder. We defined the notion of weak pseudoorders on the ordered superrings and gave some illustrative examples. Different classes of ordered hyperstructures are constructed by the properties of weak pseudoorders. In the future, we will focus on the weak pseudoorders in ordered hyperrings.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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